Quantum Relational Causality: Rank-Three Selection from a Nested-Reciprocity Source Law, and Conditional Gauge Structure

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Abstract

Quantum Relational Causality (QRC) investigates discrete relational actions on matching groupoids, treating spatial distance as the inferred measurement of causal impedance between relational excitations rather than an absolute background container. This preprint states what the program has established mathematically, under which explicit assumptions, and what remains open. The main result concerns a declared source law $S^{\mathrm{NR}}_{\mathrm{pre}}$ on a groupoid of finite sets D of excitation types with transverse matchings between D and its binary spans. For activity $0 < h < 2\lambda$ the unique ground state up to relabelling has $|D| = 3$ and the complement matching, with excitation gap $\min(3h, 2\lambda - h)$. Two further declared laws (least endpoint completion and a centred unit-rate response) then give the endpoint group $\mathbb{Z}[D]$ of rank three, exact word-ball counts $V_3(R) = (4R^3 + 6R^2 + 8R + 3)/3$, return probability $e^{-t} I_0(t/3)^3$, spectral dimension three and one co-metric $q^\sharp = \frac{1}{3}\sum_{d \in D} d \otimes d$. These are implication theorems for a declared combinatorial law; the law is not derived from informal ontological principles, and nothing here shows that nature obeys it. Separately, on a supplied Hermitian $2 + 3$ internal carrier with one determinant record, the stabilizer is $(SU(2) \times SU(3) \times U(1))/\mathbb{Z}_6$, one anomaly-free 15-state chiral family follows, and a pair-dual anomaly polynomial $(m + n - 5)\,\mathrm{ch}_3(E)$ selects the ranks $2 + 3$ within its own hypotheses; dynamical selection of the carrier, mirror removal and a joint gauge continuum are not proved. We record corrections to earlier claims, including six that failed independent audit, and fit the remaining cosmology candidates to public supernova, BAO and chronometer data: one phenomenological causal-memory dark-energy model fits at the level of ΛCDM, and the data neither prefer nor exclude it. Every numerical statement is reproducible with scripts in the accompanying package, each shown to fail on deliberately wrong input.

1 Introduction and scope

The conceptual motivation behind Quantum Relational Causality (QRC) is relational: seeking to understand whether spatial dimension, gauge symmetries, and metric structure can emerge from discrete relational configurations without assuming a prior continuous background manifold. Informally, the program explores whether the experienced three-dimensional spatial world can be understood as an assembled hierarchy of relational excitation grades (from a zero-dimensional base condition to primitive lines and spans), with time as the history of causal updates.

Crucially, this preprint does not claim to deduce physical reality from that informal metaphor, nor are the mathematical theorems derived from it. Instead, the program organises its technical proof obligations into a sequence of *gates* (Section 3), where each gate defines an explicit mathematical hypothesis and a verifiable pass condition.

This preprint has a narrow purpose: to state exactly which of those obligations have been met as mathematics, under which hypotheses, and which have not. It is written for readers who want to check the work, not for readers who want to be persuaded by it.

What is claimed.

1. For the declared source law $S^{\mathrm{NR}}_{\mathrm{pre}}$ (Definition 4.2), rank three with the complement matching is the unique ground state in the open window $0 < h < 2\lambda$, with an exact gap (Theorem 4.4). This is proved here in full and checked by exhaustive enumeration (Section 4.7).
2. Under two further declared laws (least endpoint completion and a centred response), the selected carrier has cubic word-ball growth, the stated heat kernel, spectral dimension three and a single infrared co-metric (Sections 4.3 to 4.6).
3. On a supplied internal carrier, the stated gauge group, family decomposition, anomaly cancellation and pair-dual rank selection (Section 5).

What is not claimed. That the source law follows from the foundational statement or from nonlocality; that any QRC law is realised in nature; that word length or diffusion time has been identified with measured distance or elapsed time; Lorentzian spacetime; a quantum continuum; gravity; gauge dynamics; a particle spectrum; an absolute length scale. Section 6 lists these as open problems.

Claim discipline. A theorem proved after adopting an explicit source or quantisation law is an implication theorem for that law. An implication theorem for a declared law is not evidence that nature obeys the law, and it is not evidence that more generic premises force the law. Where this paper writes "Gate 1 is passed" it means "passed as mathematics for the declared source law", which is the wording of the project status record.

Evidence labels. "Proved" means a complete proof is given in this paper. "Sketched" means the argument is outlined here and written out in a repository document named by file in Appendix A; the author's repository is not yet public, so sketched results should be read as claims until those documents are released. "Checked" means executed by a package script that is also shown failing on deliberately wrong input. "Cited" means taken from the literature, not re-derived.

2 Conceptual motivation and operational definitions

The informal conceptual heuristic that originally motivated the project is stated as:

> Space in its underlying condition is 0D and nonlocal; particle fields are 1D excitations; electromagnetic excitations are 2D; their composition supports the experienced 3D world, retaining its lower-dimensional constituents; and time is the fourth dimension encompassing its history. The limiting causal-update and propagation role is assigned to c.

The governing operational measurement definition is stated exactly:

> Distance is the inferred measurement of causal impedance. Causal impedance does not cause a separate quantity or substance called distance.

Remark 2.1 (Non-axiomatic status). The dimensional narrative above is an informal conceptual heuristic, not a mathematical premise. Nothing in this paper is mathematically deduced from the informal phrasing that particle fields are "1D" or that electromagnetism is "2D". In standard mathematical physics, physical fields are operator distributions or sections of fiber bundles over spacetime, not lower-dimensional objects composing into space. Within the formal mathematics of Section 4, the numerals $0, 1, 2, 3$ attached to relational grades denote filtration depths in an exterior matching complex, while the spatial dimension three of the conclusion is the operational exponent of a declared walk and ball count.

These are starting perspectives. Their mathematical realisation and physical adequacy are what the gates must establish. "Nonlocal" here means that spatial locality has not yet been inferred, not that a superluminal speed has been measured. No background metric, elementary graviton or universal synchronised update clock is supplied by the statement.

Response data. Let H denote a relational configuration or history and Z_H its causal-impedance response data. The type of Z_H is deliberately not fixed in advance: it is not assumed to be an electrical resistance, a scalar edge length, an update frequency or a dissipative coefficient. Gate 0 must specify its mathematical type and composition law.

Measurement definition. For a declared measurement protocol P,

$$d_P(a, b) := \mathcal{M}_P[Z_H; a, b]. \tag{1}$$

This is a measurement definition, not an equation of motion, and no particular map $\mathcal{M}_P$ has been derived. It is not permitted to choose Z_H from an independently supplied Euclidean distance and then present the recovered distance as a derivation.

Three meanings of "dimension". The formulation keeps apart (i) the dimensionality of a primitive excitation, (ii) the algebraic or form degree used to describe it and (iii) the operational spatial or spectral dimension of the assembled world; form degree alone proves neither an ontology nor the dimension of space. In Section 4 the numerals $0, 1, 2, 3$ attached to relational grades denote composition depth, and the "three" of the conclusion is an operational exponent of a declared walk and ball count. Identifying that exponent with the dimension of physical space requires the response and clock bridge of Gates 1 and 2, which is open.

3 The gate structure and current status

The gates are proof dependencies, not stages that the universe passed through in an external time.

Gate 0 specifies the nonlocal 0D condition (states, excitations, response observables, composition rules) with causal impedance defined without a prior distance. Gate 1 recovers measured three-dimensional space: cubic volume growth and spectral dimension three, with metric, measure, operator and clock identified as outputs of the declared response. Gate 2 recovers time, the causal limit c and one Lorentzian cone shared by every physical mode. Gate 3 recovers quantum preparation, evolution and measurement, including $p(i) = \mathrm{Tr}(\rho E_i)$ and the Tsirelson bound. Gate 4 recovers gravity as the matter-dependent response of causal impedance, classically and in a quantum completion. Gate 5 identifies the electromagnetic, weak and strong sectors in one interacting phase. Gate 6 derives the particle spectrum and discriminating predictions. A seventh stage collects cosmological and other observational consequences.

Table 1 reproduces the status table of the current integrated position (project status record, review completed 2026-11-19). Dated sections of that record written before this position, in particular the September 2026 entries headed "Level-4" to "Level-7" (triality generations, spectral triples, asymptotic safety, horizon-impedance dark energy, leptogenesis and others), are historical. None of them is claimed here; Section 7 reports audits of several.

The status record summarises Gates 1 and 5 with these tags:

```
MATHEMATICAL_GATE1_PASSED_FOR_DECLARED_SOURCE_LAW
EMPIRICAL_SOURCE_LAW_UNCONFIRMED
ABSOLUTE_SCALE_UNDETERMINED
CONDITIONAL_INTERNAL_GROUP_AND_ANOMALY_THEOREMS_AVAILABLE
FULL_GATE5_NOT_PASSED
```

Gates 2, 4, 5 and 6 are marked not passed. Gate 3 is marked closed only within an adopted sharp-record quantum law.

Stage	What the reviewed mathematics supports	Open physical dependency
0	Relabelling-invariant premetric interface and candidate dynamics	Selection of a source and physical history law
1	Exact rank selection under transverse reciprocity, with cubic growth and diffusion under the specified completion and walk	Source selection and operational identification of the clock and ruler
2	Exact bracket tests for the declared scalar coefficients, finite cochain certificates and conditional continuum results	One complete interacting mode inventory and a common physical cone
3	Finite record/Born construction, separated marginal tests and CHSH bound	Operational frequency law on the same positive interacting histories
4	Conditional ADM coefficient and traceless kinetic-form calculation	Positive two-helicity interacting response and universal stress coupling
5	Finite exterior family, quotient descent, anomaly cancellation and charge decomposition on a supplied internal seed	Dynamical selection, mirror removal and a joint gauge continuum
6	Allowed mass channels, exact one-family seesaw and freedom of flavour matrices	Generation count, matrix entries, vacuum and physical spectrum
7	Conditional observable calculations and a constant-equation-of-state bounce for the displayed effective law	Source-derived predictions, independent likelihoods and continuum-to-poset return map

Table 1: Gate status from the project status record (current integrated position, 2026-11-19), reproduced verbatim. "Certificate" there means a finite computation on a declared object, not an external certification.

4 Gate 1: rank selection and the cubic-diffusive phase

4.1 State space and source law

Definition 4.1 (Source groupoid)**.** An object is a pair (D, M) where

1. D is a finite unlabelled set of primitive reversible excitation types;
2. $P_2(D) = \binom{D}{2}$ is the set of all unordered binary spans (every pair is present);
3. $M \subseteq D \times P_2(D)$ is a partial matching, injective in both projections;
4. every matched pair is transverse: $(d, p) \in M \Rightarrow d \notin p$.

Morphisms are bijections of D that carry M to M. The empty object $D = \varnothing$ represents the undifferentiated 0D condition: it has no move, adjacency or distance.

No coordinate, metric, orientation, rank or length is part of an object. The relational grades $\Lambda^0, \ldots, \Lambda^3$ of $\mathbb{R}[D]$ (undifferentiated condition, primitive excitation, coupled binary span, transverse interleaving) form an increasing filtration, so every higher grade retains the lower ones; the numerals are composition depth.

Definition 4.2 (Nested-reciprocity activity law)**.** Write $n = |D|$ and $m = |M|$. The reciprocity defect is $\Delta(D, M) = (n - m) + (\binom{n}{2} - m)$, the number of unmatched primitives plus the number of unmatched spans. For $\lambda > 0$ and $h \in \mathbb{R}$,

$$S^{\mathrm{NR}}_{\mathrm{pre}}(D, M) = \lambda\, \Delta(D, M) - h\, n. \tag{2}$$

The groupoid partition function is $Z_\beta = \sum_{n \geq 0} \frac{1}{n!} \sum_M e^{-\beta S^{\mathrm{NR}}_{\mathrm{pre}}([n], M)}$, where the inner sum runs over labelled matchings on $[n]$; the factor $1/n!$ gives each isomorphism class the weight $1/|\operatorname{Aut}(D, M)|$.

This is schema `qrc.spre.nested_reciprocity_activity.v1` in the repository. The load-bearing physical assumptions of the selection result are:

(H1) every unordered binary span of excitation types is present;
(H2) matching is transverse and one-use (injective on both sides);
(H3) unmatched primitives and unmatched spans both cost a positive amount (equal costs in (2); Proposition 4.5 relaxes this);
(H4) the activity lies in the open window $0 < h < 2\lambda$.

None of (H1) to (H4) is derived from the foundational statement. The repository records explicitly that generic nonlocality does not force this law.

4.2 The selection theorem

Lemma 4.3 (Maximal transverse matching)**.** *The largest transverse matching has size* 0 *for* $n \le 2$ *and size* n *for every* $n \ge 3$*. Hence* $\min_M \Delta = 0, 1, 3$ *for* $n = 0, 1, 2$ *and* $\min_M \Delta = \binom{n}{2} - n = n(n-3)/2$ *for* $n \ge 3$*.*

Proof. For $n = 1$ there are no spans; for $n = 2$ the only span contains both elements, so transversality forbids every match. For $n \ge 3$ place D temporarily in a cyclic order $d_1, \dots, d_n$ and match $d_i \mapsto \{d_{i+1}, d_{i+2}\}$ (indices mod n). The target spans are distinct and omit their sources, so this is a transverse matching of size n. No matching is larger because the projection to D is injective. The cyclic order is discarded afterwards. Substituting m into Δ gives the minima. □

In particular $\Delta = 0$ exactly when $n = 0$, or $n = 3$ and $M = \{(d, D \setminus \{d\}) : d \in D\}$: at $n = 3$ each element is transverse only to its complementary pair, so the lossless matching is forced.

Theorem 4.4 (Rank selection and gap)**.** *The ground states of* (2) *over all objects of all ranks are: the empty object alone if* $h < 0$*; the empty object and the rank-three complement matching if* $h = 0$*; the rank-three complement matching alone, up to relabelling, if* $0 < h < 2\lambda$*. In the last case the gap to the first excited object is exactly*

$$g = \min\{3h,\ 2\lambda - h\} > 0. \tag{3}$$

Proof. Since $S_{\mathrm{pre}}^{\mathrm{NR}}$ decreases in m at fixed n, the minimum at rank $n \ge 3$ is $E_n = \frac{\lambda}{2} n(n-3) - hn$ by Lemma 4.3. Then $E_3 = -3h$ and, for $n \ge 4$,

$$E_n - E_3 = (n-3)\Big(\frac{\lambda n}{2} - h\Big), \tag{4}$$

which on $0 < h < 2\lambda$ is positive and smallest at $n = 4$, where it equals $2\lambda - h$. The empty object has energy $0 = E_3 + 3h$. Rank one has energy $\lambda - h = E_3 + \lambda + 2h$, rank two $3\lambda - 2h = E_3 + 3\lambda + h$, and removing one match from the rank-three ground state costs 2λ. Each of these is at least $\min\{3h, 2\lambda - h\}$: if $h \le \lambda/3$ then $3h \le \lambda + 2h$, and if $h \ge \lambda/3$ then $2\lambda - h \le \lambda + 2h$. For $h < 0$ every nonempty object has $S_{\mathrm{pre}}^{\mathrm{NR}} \ge -hn > 0$, and at $h = 0$ the zero-energy objects are those with $\Delta = 0$. □

Proposition 4.5 (Unequal costs)**.** *Replace* (2) *by* $\lambda_1(n-m) + \lambda_2\big(\binom{n}{2} - m\big) - hn$ *with* $\lambda_1, \lambda_2 > 0$*. For* $0 < h < 2\lambda_2$ *the rank-three complement matching is the unique ground state and the gap is* $g_{12} = \min\{3h,\ \lambda_1 + 2h,\ \lambda_1 + \lambda_2,\ 2\lambda_2 - h\}$*. The window is set by* λ_2*, the cost of unmatched spans.*

Proof (complete). At maximal matching $E_n = \frac{\lambda_2}{2} n(n-3) - hn$ for $n \ge 3$, so $E_{n+1} - E_n = \lambda_2(n-1) - h > 0$ for $n \ge 3$ on the window. The empty, rank-one, rank-two, one-missing-match and rank-four gaps are $3h$, $\lambda_1 + 2h$, $2\lambda_1 + \lambda_2 + h$, $\lambda_1 + \lambda_2$ and $2\lambda_2 - h$; the rank-two value exceeds the one-missing-match value, and every further missing match adds $\lambda_1 + \lambda_2$. □

Proposition 4.6 (The activation boundary does not select activation)**.** *At* $h = 0$ *the zero-temperature groupoid probabilities are* $\Pr(n = 3) = 1/7$ *and* $\Pr(n = 0) = 6/7$*.*

Proof. The two zero-energy classes have $|\operatorname{Aut}| = 1$ (empty) and $|\operatorname{Aut}| = |S_3| = 6$ (complement matching, which every permutation preserves). Their weights are 1 and 1/6. □

So neither zero defect nor groupoid entropy derives the activated phase; $h > 0$ is an assumption. (A repository variant inserts one abstract excitation as a boundary source; that lifts the degeneracy but explains neither the source nor a transition out of the empty sector.)

Theorem 4.7 (All-rank Gibbs concentration; sketched)**.** *$Z_\beta < \infty$ for every $\beta > 0$, and for each $\beta_0 > 0$ there is a finite C_{β_0} with $\Pr_\beta[(D, M) \ncong \text{ground}] \le C_{\beta_0} e^{-(\beta-\beta_0)g}$ for $\beta \ge \beta_0$.*

Sketch. The number of labelled partial matchings at rank n is at most $\sum_{m \le n} \binom{n}{m}\binom{\binom{n}{2}}{m} m! = e^{O(n \log n)}$, while the minimal action grows as $\lambda n^2/2$. The sum therefore converges, and every excited term carries an extra factor $e^{-(\beta-\beta_0)g}$ relative to its β_0 value. □

Remark 4.8 (Where the three comes from). No numeral three is placed in (2); it comes from (H1), since lossless reciprocity requires $n = \binom{n}{2}$. With k-ary spans the condition is $n = \binom{n}{k}$, solved by $n = k + 1$ (the check confirms that ternary spans select rank four). The result reads "binary coupling plus lossless transverse reciprocity selects three excitation types" and stands or falls with that premise. Three types are not yet a spatial dimension; that needs Sections 4.3 to 4.5.

4.3 Least endpoint completion

Definition 4.9 (Endpoint law)**.** Signed finite words in D are identified under the least congruence generated by $dd^{-1} \sim 1$ and $de \sim ed$ for $d, e \in D$ (each interchange is witnessed by the span $\{d, e\}$). "Least" means that no other identification is imposed.

The resulting endpoint group is the universal abelian group on D: $A(D) = \mathbb{Z}[D]$, free abelian of rank $|D|$. Equivalently, rotor characters $D \to SO(2)$ separate distinct exponent vectors. Relational positions form an $A(D)$-torsor, so no origin is chosen. The word distance is $r_D(x, y) = \sum_d |k_d|$ for $y - x = \sum_d k_d[d]$; it is invariant under translation, reversal of each primitive and every relabelling. Finite regulators are the characteristic quotients $Q_m = A(D)/mA(D)$, which choose no axes and have word systole exactly m.

The completion clause is load bearing, and each alternative is a countermodel: the extra relation $\sum_d d = 0$ reduces the rank to two and gives quadratic growth; omitting interchange gives the free non-abelian group and exponential growth; letting the three types act trivially gives no extended space at all.

4.4 Word-ball growth

Proposition 4.10 (Exact ball count)**.** *For $|D| = n$, the number of elements of $A(D)$ within word radius R is $V_n(R) = \sum_{j=0}^{n} 2^j \binom{n}{j}\binom{R}{j}$. In the selected phase*

$$V_3(R) = \frac{4R^3 + 6R^2 + 8R + 3}{3} \sim \frac{4}{3}R^3, \tag{5}$$

and on Q_m the same count holds exactly for $2R < m$.

Proof. Choose the j nonzero coefficients ($\binom{n}{j}$ ways), their signs (2^j) and positive magnitudes with sum at most R ($\binom{R}{j}$ ways). Summing for $n = 3$ gives (5). On Q_m the quotient map is injective on balls of radius R when the systole m exceeds $2R$. □

Equation (5) is the standard lattice-point count of the octahedron $|x| + |y| + |z| \le R$ in $\mathbb{Z}^3$; the mathematics is classical, and the content of Gate 1 lies in the chain that leads to it. Two limits on what it shows: no bounded-radius test can certify asymptotic dimension (the map $\mathbb{Z}^4 \to \mathbb{Z}^3$, $e_4 \mapsto \sum_i N_i e_i$ with $1 + \sum_i |N_i| > 2R$, is injective on every radius-R ball), and a constant physical length per step, if one is later assigned, changes only the coefficient 4/3.

4.5 Centred response, heat kernel and spectral dimension

Declare the response generator to be (R1) conservative and translation invariant, (R2) invariant under reversal of each primitive and under relabelling, (R3) supported on single primitive moves $\pm[d]$, (R4) free of killing, drift and long jumps, and (R5) normalised to total jump rate one. The empty object has generator 0 and return probability 1.

Proposition 4.11 (Unique generator)**.** *For nonempty D, (R1) to (R5) give exactly*

$$L_D f(x) = \frac{1}{2|D|} \sum_{d \in D} \big[f(x + [d]) + f(x - [d]) - 2f(x) \big]. \tag{6}$$

Proof. By (R1) and (R3) the generator is determined by rates $r_{\pm d}$. Reversal invariance gives $r_{+d} = r_{-d}$, relabelling invariance makes all rates equal, and (R5) fixes the common value $1/(2|D|)$. □

(R1) to (R5) are a declared clock class; the parameter t below is a dimensionless diffusion parameter, not physical time (Gate 2).

Proposition 4.12 (Return probability)**.** *On the infinite cover,*

$$p_D(t; 0, 0) = e^{-t}\, I_0(t/|D|)^{|D|}\,. \tag{7}$$

Proof. Under (6) the $|D|$ coefficients evolve as independent continuous-time symmetric walks on $\mathbb{Z}$, each with total jump rate $1/|D|$. A symmetric walk on $\mathbb{Z}$ with total rate ν returns with probability $e^{-\nu t} I_0(\nu t)$; take the product. □

Theorem 4.13 (Spectral dimension three)**.** *For $|D| = 3$,*

$$d_s(t) := -2t \frac{d}{dt} \log p_D(t; 0, 0) = 2t \Big[1 - \frac{I_1(t/3)}{I_0(t/3)} \Big] \;\longrightarrow\; 3 \qquad (t \to \infty). \tag{8}$$

Proof. Differentiate (7) using $I_0' = I_1$. The large-argument expansion $1 - I_1(x)/I_0(x) = \frac{1}{2x} + O(x^{-2})$ with $x = t/3$ gives $d_s(t) = 3 + O(1/t)$. The logarithmic definition $-2 \log p / \log t$ also tends to 3, since $p_D \sim (3/2\pi t)^{3/2}$. □

Finite regulators. On Q_m a return that is not a return on the cover needs a displacement in $mA(D) \setminus \{0\}$, hence at least m jumps. So $|p_m(t) - p_\infty(t)| \le \Pr\{\mathrm{Pois}(t) \ge m\}$, and termwise differentiation gives $|p_m'(t) - p_\infty'(t)| \le 2 \Pr\{\mathrm{Pois}(t) \ge m - 1\}$. Along $t_m = \sqrt{m}$ both are superpolynomially small compared with $t_m^{-3/2}$, so $d_{s,m}(t_m) \to 3$. A fixed finite quotient eventually mixes and has late-time spectral dimension 0; the cofinal order of limits is part of the statement.

Joint source and heat limit (sketched). Instead of selecting the ground state first, take $t_m = \sqrt{m}$ and $\beta_m = \beta_0 + \frac{2}{g} \log m$. Theorem 4.7 gives $\Pr_{\beta_m}(\text{not ground}) = O(m^{-2}) = O(t_m^{-4})$. Every nonempty normalised generator has return in $[0, 1]$ and a bounded time derivative, and the empty generator has return 1 and derivative 0, so the annealed derivative spectral dimension, taken at fixed (β, m) and then evaluated on the diagonal, also tends to 3. The integer window $R_m = \lfloor \sqrt{m} \rfloor$ stays prewrap, so (5) holds there with probability tending to one.

4.6 The infrared co-metric

Proposition 4.14 (Acoustic co-metric)**.** *For a character $\theta \in \mathrm{Hom}(A(D), \mathbb{R})$ the Bloch eigenvalue of $-L_D$ at $|D| = 3$ is*

$$\varepsilon_D(\theta) = \frac{1}{3}\sum_{d\in D}(1-\cos\theta(d)) = \frac{1}{2}\,q^\sharp(\theta,\theta) + O(|\theta|^4), \qquad q^\sharp = \frac{1}{3}\sum_{d\in D} d\otimes d. \tag{9}$$

Proof. Apply (6) to $e^{i\theta(x)}$ and expand $1-\cos u = u^2/2 + O(u^4)$. □

The tensor $q^\sharp$ is coordinate free and invariant under relabelling and reversal. Three elementary consequences: (i) in the one-vertex periodic cell each unoriented edge pair has conductance $1/3$ and the corrector incidence vanishes, so the effective tensor is $A_{\mathrm{eff}} = q^\sharp$, and $K_q = -2L_D$ has principal symbol $q^\sharp$; this fixes the factor of two between the Markov clock and the wave normalisation, so word distance, volume, diffusion and the scalar seed use one metric. (ii) The q-ball count is $\#\{a : q(a,a) \le R^2\} = \frac{4\pi}{9\sqrt{3}}R^3 + O(R^2)$, with $q = (q^\sharp)^{-1}$. (iii) A scalar Hamiltonian built on c^2K_q has infrared dispersion $\omega^2 = c^2q^\sharp(k,k) + m^2 + O(|k|^4)$. Item (iii) is only a principal seed for Gate 2: it does not show that every physical species shares this symbol, and $q^\sharp$ selects an $O(3)$ frame orbit, not an oriented coframe or a Lorentzian history.

4.7 Numerical verification

Two scripts check the Gate 1 statements. Each records an expected outcome for every test, including tests on deliberately wrong inputs that must fail, and exits with status 0 only if every observed outcome matches its expectation. Table 2 summarises them; full output is in the package.

Statement	Independent computation	Result
Theorem 4.4, Proposition 4.5	Exhaustive enumeration of every labelled object for $n \le 6$ (772,621 objects at $n = 6$), exact rational arithmetic, 11 parameter points; bound $m \le n$ for $6 < n \le 400$	Unique ground state is the rank-three complement matching, $\lvert\,\mathrm{Aut}\,\rvert = 6$; gaps equal (3) and g_{12} exactly
Lemma 4.3	Enumeration ($n \le 6$), augmenting-path matching ($n \le 12$)	max $m = 0, 0, 0, 3, 4, \ldots, 12$
Enumerator	Independent bitmask dynamic programme	counts agree for $n \le 6$
Proposition 4.6	Groupoid weights of the zero-energy classes	$\Pr(n = 3) = 1/7$ exactly
Equation (5)	Breadth-first search on the Cayley graph of $\mathbb{Z}[D]$, $R \le 30$; torus Q_{15}	exact; fails after wrap ($R > 7$) as stated
Equation (7)	Poissonised multinomial series; $\exp(tL_D)$ on a 41^3 torus	relative error 3×10^{-15} and 8×10^{-16}
Theorem 4.13	Finite-difference derivative of $\log p$	$d_s = 3.362$, 3.023, 3.002, 3.0002, 3.00002 at $t = 10$ to 10^5
Equation (9)	Symbol of L_D on plane waves, random non-orthogonal realisation of D	relative error 1.6×10^{-7}

Table 2: Gate 1 checks. Rows 1 to 4: `check_gate1_selection.py` (47 tests, 14 of them on wrong inputs). Rows 5 to 8: `check_gate_results.py` (from an independent audit; 26 tests, 13 on wrong inputs, covering Gate 5 as well).

The wrong inputs, all of which fail as required, are: $h = 2\lambda$, $h > 2\lambda$, $h = 0$ and $h < 0$; dropping transversality (six labelled lossless matchings at rank three); ternary spans (rank four selected); no cost

for unmatched spans (no minimum); a window set by λ_1 instead of λ_2; wrong gap formulas; labelled instead of groupoid weights; and an enumerator without span-side injectivity.

4.8 What Gate 1 does not establish

1. The source law (H1) to (H4), the endpoint law and the response class (R1) to (R5) are declared, not derived. The repository states that generic nonlocality does not force them.
2. The selected "three" is the rank of an endpoint group and the exponent of a declared ball count and walk. Identifying word length with measured distance and the diffusion parameter with elapsed time requires a protocol $\mathcal{M}_P$ in (1) and a clock, which are open (Section 6).
3. Positive activity is assumed (Proposition 4.6).
4. No absolute length is determined, and none is needed for the exponents.
5. The repository's complete Gate 1 contract (schema version 2, "refinement stationary") replaces the characteristic quotients by random prime-cyclic regulators with a Haar-distributed quenched response. Its additional premises (strong monoidality and split-and-recombine stationarity of the response, quenched response, independent additive gluing of singleton attempt clocks) are load bearing, are not derived from the rank action, and are not reviewed here. The characteristic-quotient construction above is an explicit witness under its own stated assumptions.

5 Gate 5: conditional internal-group and anomaly results

Everything in this section is conditional on a supplied internal carrier. Nothing here derives that carrier from Gate 1 or from any earlier QRC structure, and nothing here establishes a gauge phase.

5.1 The internal-spacetime type wall

Let $\mathcal{A}_{\mathrm{st}}$ be the algebra generated by spacetime propagation, clock and frame operations on a low-energy state space $\mathcal{H}$. The internal algebra is its commutant $\mathcal{A}'_{\mathrm{st}}$; for $\mathcal{H} = \bigoplus_\lambda V_\lambda \otimes M_\lambda$ with $\mathcal{A}_{\mathrm{st}} = \bigoplus_\lambda \mathcal{B}(V_\lambda) \otimes 1$ it is $\bigoplus_\lambda 1 \otimes \mathcal{B}(M_\lambda)$.

Proposition 5.1 (Non-identifiability of internal multiplicity)**.** *The replacement* $(\mathcal{H}, \mathcal{A}_{\mathrm{st}}) \mapsto (\mathcal{H} \otimes \mathbb{C}^r, \mathcal{A}_{\mathrm{st}} \otimes 1_r)$ *leaves every spacetime observable unchanged and multiplies the commutant by* $M_r(\mathbb{C})$*. Hence spatial rank three and the nested spatial grades do not determine internal ranks.*

The proof is the statement itself: expectation values of $\mathcal{A}_{\mathrm{st}} \otimes 1_r$ in states $\psi \otimes \chi$ do not depend on r. This is why the internal carrier below must be supplied.

5.2 Supplied seed and stabilizer

Hypotheses. (G1) A Hermitian complex space $E \subset \mathcal{A}'_{\mathrm{st}}$ with $\dim_{\mathbb{C}} E = 5$. (G2) An orthogonal projector P of rank two, giving $E = W \oplus C$ with $W = \operatorname{im} P$, $C = \ker P$. (G3) One unit vector $\Omega \in \det E$, including its phase. (G4) No other invariant internal tensor, and local frame changes preserving (P, Ω) are gauge redundancies.

Theorem 5.2 (Stabilizer)**.** *The unitary stabilizer of* (P, Ω) *is* $G = S(U(2) \times U(3)) = \{(u_2, u_3) : \det u_2 \det u_3 = 1\}$*. The map* $\phi(g_2, g_3, z) = (z^3 g_2, z^{-2} g_3)$ *from* $SU(2) \times SU(3) \times U(1)$ *is surjective with kernel generated by* $(-I_2, \omega I_3, e^{i\pi/3})$*,* $\omega = e^{2\pi i/3}$*, of order six, so*

$$G \cong \frac{SU(2) \times SU(3) \times U(1)}{\mathbb{Z}_6}. \tag{10}$$

Proof. Preserving P means preserving the splitting, giving $U(2) \times U(3)$; the action on $\det E$ is multiplication by $\det u_2 \det u_3$, and fixing Ω imposes the determinant condition. Given $(u_2, u_3) \in G$ pick z with $z^6 = \det u_2$; then $g_2 = z^{-3}u_2$ and $g_3 = z^2 u_3$ have unit determinant, so ϕ is onto. The kernel requires $z^3 g_2 = I$ and $z^{-2} g_3 = I$, so g_2, g_3 are central and $z^6 = 1$, which gives the stated cyclic group of order six. □

The hypercharge generator is $Y = \frac{1}{2}P - \frac{1}{3}(1 - P)$, which acts on $\Lambda^p W \otimes \Lambda^q C$ as $Y = p/2 - q/3$.

5.3 One anomaly-free chiral family

Theorem 5.3 (Exterior family)**.** *Under $SU(3) \times SU(2) \times U(1)_Y$,*

$$\Lambda^2 E \oplus \Lambda^4 E = (\mathbf{3}, \mathbf{2})_{1/6} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-2/3} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{1/3} \oplus (\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{1}, \mathbf{1})_1, \tag{11}$$

fifteen left-handed Weyl states with the quantum numbers of Q, u^c, d^c, L, e^c.

Proof. $\Lambda^2 E = \Lambda^2 W \oplus (W \otimes C) \oplus \Lambda^2 C$ gives $(\mathbf{1}, \mathbf{1})_1$, $(\mathbf{3}, \mathbf{2})_{1/6}$, $(\bar{\mathbf{3}}, \mathbf{1})_{-2/3}$; $\Lambda^4 E = (W \otimes \Lambda^3 C) \oplus (\Lambda^2 W \otimes \Lambda^2 C)$ gives $(\mathbf{1}, \mathbf{2})_{-1/2}$ and $(\bar{\mathbf{3}}, \mathbf{1})_{1/3}$, using $Y = p/2 - q/3$ and $\Lambda^2 \mathbf{3} = \bar{\mathbf{3}}$. □

Adding $\Lambda^0 E$ gives a neutral singlet with the quantum numbers of a sterile ν^c (sixteen states). Neither the group nor anomaly cancellation forces or forbids it. The physical realisation of this fermionic module, its chirality and its multiplicity one are hypotheses.

Proposition 5.4 (Anomalies; checked)**.** *With $T(\mathbf{3}) = T(\mathbf{2}) = \frac{1}{2}$ and all fields left handed, the coefficients $SU(3)^3$, $SU(3)^2 Y$, $SU(2)^2 Y$, $\mathrm{grav}^2 Y$ and Y^3 are $2 - 1 - 1$, $\frac{1}{6} - \frac{1}{3} + \frac{1}{6}$, $\frac{1}{4} - \frac{1}{4}$, $1 - 2 + 1 - 1 + 1$ and $(1 - 32 + 4 - 9 + 36)/36$, all zero. There are four weak doublets, so the Witten $SU(2)$ anomaly [16] cancels. Every multiplet satisfies $3s + 2t + 6Y \equiv 0 \pmod 6$ (s the doublet parity, t the triality), so it descends to the $\mathbb{Z}_6$ quotient.*

That the quotient group has no further four-dimensional spin global anomaly once the local conditions hold is cited from the bordism computation of Davighi, Gripaios and Lohitsiri [4] and is not checked here. Anomaly freedom of the representation does not construct a fermion measure.

5.4 The pair-dual rank selector

Theorem 5.5 (Pair-dual anomaly selector; sketched and checked)**.** *Let $E = A_m \oplus B_n$ with $m, n \geq 2$, $N = m + n$, one combined determinant record (so $c_1(E) = 0$) and stabilizer $S(U(m) \times U(n))$. Assume one nonzero net-chirality copy of $F_N = \Lambda^2 E \oplus E^*$ and no other chiral cancellation sector, Green-Schwarz or Wess-Zumino term, or inflow. Then*

$$I_6(F_N) = (N - 5)\ \mathrm{ch}_3(E), \tag{12}$$

which vanishes identically if and only if $N = 5$; with $m, n \geq 2$ this forces $\{m, n\} = \{2, 3\}$.

Sketch. From $\mathrm{ch}(\Lambda^2 E) = \frac{1}{2}[(\mathrm{ch}\, E)^2 - \mathrm{ch}(\Psi^2 E)]$ and $\mathrm{ch}_1(E) = 0$, the degree-three part is $\mathrm{ch}_3(\Lambda^2 E) = \frac{1}{2}(2N - 8)\,\mathrm{ch}_3(E) = (N - 4)\,\mathrm{ch}_3(E)$, and $\mathrm{ch}_3(E^*) = -\mathrm{ch}_3(E)$. For $m, n \geq 2$, $\mathrm{ch}_3(E)$ is not the zero polynomial: if a pure cubic term vanishes, a mixed term proportional to $f\,\mathrm{tr}\, F_m^2$ remains. The case $(2, 2)$ also has three doublets per $SU(2)$ factor and fails the Witten test. □

The identity is checked numerically for $(m, n) \in \{(2, 2), (2, 3), (3, 3), (2, 4), (3, 4), (4, 4)\}$ with explicit Λ^2 matrices on random traceless block curvatures (Appendix A). The selector moves the question rather than answering it: it does not derive two blocks, the determinant record or the pair-dual package.

5.5 Higgs channel

Requiring the three charged Yukawa channels Qu^cH, Qd^cH^* and Le^cH^* forces a weak doublet H with $Y_H = \frac{1}{2}$. A nonzero neutral vector of H has stabilizer $S(U(1) \times U(3)) \cong U(3) \cong (SU(3) \times U(1)_{\rm em})/\mathbb{Z}_3$ with unbroken generator $Q = T_3 + Y$, and with a positive gauge kinetic term Q is the unique tree-level massless electroweak direction. This is representation theory. It does not determine the sign of the Higgs mass term, a Higgs phase, Yukawa values, or a physical photon.

At one loop, N_g families and one Higgs doublet give $(B_3, B_2, B_Y) = (-11 + \frac{4}{3}N_g, \frac{-43+8N_g}{6}, \frac{40N_g+3}{18})$ (checked from the field content). These signs are perturbative flow polarity only. The group and matter do not force equal couplings; $\sin^2\theta_W = 3/8$ follows only if an extra full-carrier trace law sets $g_3 = g_2 = g_1$ at some scale.

5.6 What is not proved in Gate 5

1. Derivation of the seed (G1) to (G4) from the physical operator algebra of earlier gates; Proposition 5.1 shows spacetime observables cannot do it.
2. A finite-regulator measure that is simultaneously one-family chiral, local on the required rough colour support, quotient-global, reflection positive and cofinally compatible. The repository proves a same-cutoff obstruction: the rank of a continuous exact-index projector is constant on the connected space of unrestricted links, while the index must change between topological sectors.
3. Removal of the mirror sector with a uniform relative gap.
4. A physical photon: a positive-residue massless pole with two helicities and a nonzero charge vertex.
5. Colour confinement with string breaking and no coloured asymptotic states, in a continuum limit. Fixed-cutoff strong coupling does not give this.
6. One joint measure realising all of these together. Separate sector results do not compose, because the chiral determinant and the $\mathbb{Z}_6$ flux class couple all factors.
7. Family number, Higgs uniqueness, Yukawa couplings and masses (Gate 6).

The repository states a conditional theorem that derives the stated family, one interacting two-helicity photon, screened $SU(3)$ confinement and no mirror family from a cofinal family of local G Hamiltonians satisfying all of these requirements uniformly. It is an implication whose hypotheses have not been constructed.

6 Open problems

Gate 0: selection of the source law. No argument selects $S^{\rm NR}_{\rm pre}$, the endpoint law or the response class from the bare relational condition. Generic nonlocality does not force them. This is the first unproved implication in the chain.

Gate 1 bridge: ruler and clock. The measurement definition (1) has not been instantiated. Repository experiments on the native source give partial results (a direct incidence readout that distinguishes launches a scalar mediator cannot, an embedded six-way walk extracted from shortest returns of a marked triad, a passive output instrument with a finite sampling budget), but equal response-time laws at different graph separations prevent the tested protocol from assigning distinct ranges. Physical sensors, a co-located marker, an independently calibrated length and a common laboratory clock are open; no physical Gate 1 range has been obtained.

Gate 2: one causal cone. Finite and free results exist (finite marked-cut brackets, free mode inventories, a scalar principal seed). Static Gate 1 data cannot fix a temporal structure: an explicit two-species family preserves every Gate 1 datum while carrying unequal limiting speeds, and reversible

Markov time cannot be transformed into hyperbolic Hamiltonian evolution. A complete interacting mode inventory with one common cone, controlled refinement and reflection positivity is not constructed.

Gate 3: quantum rules. Within an adopted sharp-record Clifford-circuit law the repository derives Born probabilities, separated no-signalling and Tsirelson saturation; the prelocal law does not select this quantum extension, and its interacting continuum carrier depends on Gate 2.

Gate 4: gravity. Under one-metric hypersurface-deformation hypotheses the nonlinear ADM action and two massless helicities are reconstructed. A positive interacting graviton continuum, positive residue, absence of extra poles and a same-metric matter Ward identity are open. Inserting a free transverse-traceless block would not derive the matter-to-impedance mechanism.

Gate 6: spectrum. Conditional on the $2+3$ carrier, exterior occupation gives the charge types and allowed mass channels. Amplification and Yukawa countermodels show that the preceding gates do not determine the number of generations, masses, hierarchies, mixing matrices, CP phases or the Dirac versus Majorana branch.

Gate 5, observations and scale. For Gate 5 see the end of Section 5. Source-derived cosmological predictions, independent likelihoods and a map from continuum observables back to the relational poset are open; the models of Section 8 are not derived from the source law. Gate 1 neither needs nor determines a physical length per step or a time per unit of diffusion parameter.

Empirical status of the source law. The premises (H1) to (H4) are falsifiable in principle, but no observable consequence that distinguishes them from standard physics has yet been derived. Producing one is a precondition for the source law to count as physics rather than as a mathematical model.

7 Corrections and negative results

The program has made claims that did not survive checking. Two independent audits run on 2026-11-22 (one of cosmology scripts, one of other numerical claims) recomputed them from scratch. Their findings are part of the research record and are reported here as corrections. The policy is to withdraw a failed identification or mechanism and to return the question to the QRC obligation it was meant to discharge, not to substitute a standard result for it. Observations remain the test throughout.

1. **The identification $Z_H = c^3/G$ is withdrawn as underived.** Earlier documents assert $Z_H = \delta\Pi_{ab}/\delta\sigma^{ab} = c^3/G$ in one line, with no intermediate step, using H for a horizon rather than a relational history. $c^3/G \approx 4.04 \times 10^{35}\ \mathrm{kg\,s^{-1}}$ is the Planck mass divided by the Planck time, a unit rather than a derived response. The step was also internally inconsistent: it started from the Einstein-Hilbert and Gibbons-Hawking-York action, and with that same action the horizon shear response is $c^3/(8\pi G)$ [13, 3], not c^3/G. QRC adopts neither value. The identification conflicts with QRC's own measurement definition (Section 2): it fixes the type of Z_H as a dissipative scalar, presupposes G, c, a smooth horizon and Einstein dynamics (outputs of Gates 2 and 4, which are not passed), and fixes a Planck scale while the absolute scale is undetermined. Under that definition the type, composition law and value of Z_H remain open; specifying them is the Gate 0 obligation. The bound $P_{\mathrm{GW}} \le c^5/G = Z_H c^2$ is a pre-existing conjecture and is withdrawn as a QRC output.
2. **The horizon-impedance (holographic) dark-energy mechanism is withdrawn.** The proposal $E = TS = c^5/(2GH)$ divided by the Hubble volume gives $3c^2H^2/(8\pi G) = \rho_{\mathrm{crit}}c^2$ exactly, so $\Omega_{\mathrm{DE}} = 1$: a restatement of the Friedmann equation, not a prediction. Against the Planck value

$\Omega_\Lambda = 0.6847 \pm 0.0073$ [11] it is a factor 1.46 too large. The claimed resolution of the 10^{122} problem is the identity $(3/8\pi)(R_H/\ell_P)^2$. Any law $\rho_{\rm DE} \propto H^2$ with coefficient below one gives $H \propto a^{-3/2}$ [6], which the data reject (Section 8).

3. **The causal-boundary expansion law ($H_0 = 1/t_0$, $R_h = ct$) is withdrawn.** The value $977.79/13.787 = 70.92\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ uses $t_0 = 13.787$ Gyr, the Planck ΛCDM age, which is a fitted output; in ΛCDM $H_0 t_0 \approx 0.951$, so the input fixes the output and the result is not parameter-free. Taken as an expansion law, $H = 1/t$ is the coasting $R_h = ct$ history [7], which the data reject (Section 8); within it the chronometers give $H_0 = 61.2 \pm 1.7$, and $H_0 = 70.92$ gives $\chi^2 = 42.0$ for 15 points. The constant 0.22 mag void backreaction proposed alongside it is absorbed by the supernova absolute magnitude and is withdrawn with it.
4. **The agegraphic dark-energy route has the wrong-sign drift.** For the agegraphic model of Wei and Cai [15] solved self-consistently with $\Omega_{\rm DE,0} = 0.685$: $n = 2.558$, $w_0 = -0.784$, $w_a = +0.136$. The sign is structural: w goes from $-1/3$ in the radiation era to about $-2/3$ in the matter era and then decreases towards -1, so $w_a = -dw/da > 0$. DESI combinations prefer $w_a < 0$ [5]. The earlier script computed $w_a = +0.20$ yet passed a "$w_a < 0$" claim because it tested only $|w_a| < 1$. The "parameter-free" coupling $n = 1$ forces $\Omega_m \approx 0.79$ and is withdrawn; with n fitted the route survives only as a mildly disfavoured candidate (Section 8).
5. **PSR J0337+1715 does not distinguish QRC from general relativity.** QRC documents assert a strong-equivalence-principle parameter $\Delta = 0$ exactly, which is the general-relativistic value and is consistent with the published bounds $|\Delta| < 2.6 \times 10^{-6}$ [1] and $|\Delta| < 1.8 \times 10^{-6}$ [14]. The accompanying "unitarity proof" returns whatever Δ its ansatz contains, and the "forward walk" estimator, run on post-fit residuals in which Δ was already fitted, returns $\Delta \approx 0$ even on synthetic data generated with $\Delta = 5 \times 10^{-6}$.
6. **Earlier multi-survey "confirmations" used no catalogue data.** All five datasets in that script were hand-typed tables with diagonal errors: "Pantheon+" was 17 rows, departing from the binned real data by up to 0.20 mag with errors three to five times the real bin errors; "quasars" was 14 rows; the DESI table had misplaced entries. The quoted reduced χ^2 values (0.259 for QRC against 1.483 for ΛCDM on Pantheon+) are not printed by the current script and were not reproduced by 96 variants. On the real Pantheon+ data with full covariance, ΛCDM fits better than the script's QRC model ($\Delta\chi^2 = +10.6$).

What remains of the cosmology. With items 2 and 3 withdrawn and item 4 weakened, the surviving QRC cosmological route is the causal-memory kernel of Section 8, in which dark energy is a retarded response to the matter content of the past. Its earlier derivation from the conformal trace anomaly fails (below), so a QRC-native derivation of the kernel, its source and its relaxation time is open; until one exists the kernel is phenomenological.

Further corrections. The retarded-trace model's $w_a = -0.227$ was a one-sided finite-difference artefact (converged -0.14 as written, -0.18 self-consistent); its trace-anomaly mechanism fails because $z = 1100$ is matter dominated; its early-dark-energy bound holds for any non-negative source and so cannot fail. The September 2026 strong-CP and baryon-asymmetry claims fail as tested (the determinant phase is π for 91 of 200 random seeds; with the standard efficiency coefficient the asymmetry is 1.16×10^{-10}, not 6.12×10^{-10}). The unimodular-gravity scripts restate known physics and do not predict Λ. The 2026-11-18 dependency audit found that a proposed formula for $1/\alpha$ gives 137.0360824, about 3965 quoted standard uncertainties from the CODATA 2022 value 137.035999177(21), and that a one-loop $\Lambda_{\rm QCD}$ claim of 0.332 GeV contradicts its own equation, which gives 6.46×10^{11} GeV. Applied-physics simulations in the archive (navigation, a decihertz clock telescope, Casimir forces) contained hard-coded results and are not part of this preprint.

8 Cosmology candidates against data

The cosmology models below are motivated by QRC language about causal memory and the past light cone, but none is derived from the Gate 1 source law. They are tested as phenomenological expansion laws. The withdrawn mechanisms of Section 7 ($R_h = ct$, the constant backreaction, the H^2 law, the $n = 1$ agegraphic coupling) are included to show the data behind their withdrawal; the candidates still open are the causal-memory kernel and, more weakly, the agegraphic model with n fitted.

Harness validation. Before any QRC model was fitted, the harness reproduced published flat-ΛCDM results with tolerances fixed before the first run: Pantheon+ alone gives $\Omega_m = 0.3331 \pm 0.0183$ (published 0.334 ± 0.018 [2]); Pantheon+ with SH0ES calibrators gives $H_0 = 73.53 \pm 1.02$ (published 73.6 ± 1.1); DESI DR1 BAO gives $\Omega_m = 0.2946 \pm 0.0146$ and $hr_d = 101.90 \pm 1.27$ Mpc (published 0.295 ± 0.015 and 101.8 ± 1.3 [5]); DES-SN5YR Dovekie gives $\Omega_m = 0.3307 \pm 0.0153$ (published 0.330 ± 0.015 [12]). As a negative control, Einstein-de Sitter is rejected with $\Delta\chi^2 = +693$ (Pantheon+), $+927$ (DES) and $+482$ (DESI). Five deliberately wrong harness variants were run: four were caught; the fifth (applying the redshift cut to the inverse covariance instead of the covariance) shifts Ω_m by only 0.002, below tolerance, so it was not caught, and the correct ordering was confirmed by reading the code.

Data and method. Pantheon+ (1580 Hubble-flow supernovae, full statistical and systematic covariance, M profiled) [2]; DES-SN5YR Dovekie (1820 supernovae, offset profiled) [12]; DESI DR1 BAO (12 numbers, full covariance, hr_d profiled) [5]; 15 cosmic-chronometer points (BC03, diagonal errors, h profiled) [8, 9, 10]. Pantheon+ and DES are never combined because they share supernovae. Ω_r is fixed and neutrinos are massless. k counts cosmological plus nuisance parameters, $\mathrm{AIC} = \chi^2 + 2k$, $\mathrm{BIC} = \chi^2 + k \ln N$, both relative to ΛCDM on the same data. No CMB likelihood is used.

The causal-memory model. The harness implements

$$\rho_{\mathrm{DE}}(t) = K \int_0^t e^{-(t-t')/\tau}\, S(t')\, dt', \qquad S = \frac{\rho_m}{\rho_m + 2\rho_r}\sqrt{\rho_m}, \tag{13}$$

with K fixed by $\Omega_{\mathrm{DE},0}$ and τ in units of $1/H_0$, solved either on a ΛCDM background ("as written") or self-consistently. Its proposed derivation from the conformal trace anomaly does not hold (Section 7), so the kernel is phenomenological. The value $\tau = 1.35$ was chosen by hand, not derived, and it sets w_0; with τ free, τ is fitted. On the earlier ΛCDM background w crosses -1 near $z \approx 2.6$, so the history is phantom at earlier times.

Results by model. The agegraphic model with n free is mildly disfavoured ($\Delta\chi^2 \approx +4$ at equal k) and has the wrong-sign drift; in the CPL plane with Ω_m profiled its (w_0, w_a) point lies $\Delta\chi^2 = 7.9$ (Pantheon+) and 10.4 (DES) above the CPL best fit. With $n = 1$ it forces $\Omega_m \approx 0.79$ and is rejected. $R_h = ct$ is rejected (DESI alone gives $\chi^2 = 83.1$ for 12 points even with hr_d free, which had to be treated as a free ruler because the sound horizon diverges logarithmically when $a \propto t$); the constant 0.22 mag backreaction is absorbed by the supernova absolute magnitude and changes nothing. Within $R_h = ct$ the chronometers give $H_0 = 61.2 \pm 1.7$ while SH0ES-calibrated supernovae give 71.6 ± 1.0. The H^2 law with coefficient one is the Friedmann equation and not testable as written; its only testable generalisation is rejected. The causal-memory model with $\tau = 1.35$ has $w_0 \approx -0.80$, $w_a \approx -0.17$ and $\Delta\mathrm{AIC} = +1.7$ (Pantheon+) and $+0.2$ (DES); with τ free, $\Delta\mathrm{AIC} = -1.1$ and -2.1 but $\Delta\mathrm{BIC} = +4.2$ and $+3.4$. For comparison, CPL itself lands in the DESI quadrant ($w_0 > -1$, $w_a < 0$) with a preference over ΛCDM of about 1.3σ to 2.1σ for two extra parameters, and BIC prefers ΛCDM.

Model	k	Best fit	χ^2	$\Delta\chi^2$	ΔAIC	ΔBIC
Pantheon+ + DESI DR1 + CC ($N = 1607$)						
ΛCDM	4	$\Omega_m = 0.310$	1408.1	0	0	0
CPL w_0w_aCDM	6	$\Omega_m = 0.311$, $w_0 = -0.876$, $w_a = -0.454$	1404.7	-3.4	$+0.6$	$+11.3$
Agegraphic, n free	4	$n = 2.85$ ($\Omega_m = 0.261$), $w_0 = -0.80$, $w_a = +0.14$	1412.5	$+4.4$	$+4.4$	$+4.4$
Agegraphic, $n = 1$	3	$\Omega_m = 0.788$ (derived)	2190.1	$+782.0$	$+780.0$	$+774.6$
$R_h = ct$	3	none	1581.3	$+173.2$	$+171.2$	$+165.8$
$R_h = ct$ + 0.22 mag	3	none	1581.3	$+173.2$	$+171.2$	$+165.8$
Holographic H^2 law	3	none ($H \propto a^{-3/2}$)	2596.7	$+1188.6$	$+1186.6$	$+1181.2$
Memory, $\tau = 1.35$	4	$\Omega_m = 0.292$, $w_0 = -0.80$, $w_a = -0.17$	1409.8	$+1.7$	$+1.7$	$+1.7$
Memory, τ free	5	$\Omega_m = 0.302$, $\tau = 2.58$, $w_0 = -0.91$, $w_a = -0.10$	1405.0	-3.1	-1.1	$+4.2$
DES-SN5YR Dovekie + DESI DR1 + CC ($N = 1847$)						
ΛCDM	4	$\Omega_m = 0.313$	1653.3	0	0	0
CPL w_0w_aCDM	6	$\Omega_m = 0.323$, $w_0 = -0.798$, $w_a = -0.999$	1647.2	-6.1	-2.1	$+9.0$
Agegraphic, n free	4	$n = 2.87$ ($\Omega_m = 0.257$), $w_0 = -0.80$, $w_a = +0.14$	1657.5	$+4.2$	$+4.2$	$+4.2$
Agegraphic, $n = 1$	3	$\Omega_m = 0.788$ (derived)	2589.8	$+936.5$	$+934.5$	$+929.0$
$R_h = ct$	3	none	1850.6	$+197.3$	$+195.3$	$+189.8$
$R_h = ct$ + 0.22 mag	3	none	1850.6	$+197.3$	$+195.3$	$+189.8$
Holographic H^2 law	3	none ($H \propto a^{-3/2}$)	3075.6	$+1422.3$	$+1420.3$	$+1414.7$
Memory, $\tau = 1.35$	4	$\Omega_m = 0.290$, $w_0 = -0.80$, $w_a = -0.17$	1653.5	$+0.2$	$+0.2$	$+0.2$
Memory, τ free	5	$\Omega_m = 0.301$, $\tau = 2.38$, $w_0 = -0.90$, $w_a = -0.10$	1649.2	-4.1	-2.1	$+3.4$

Table 3: Fits to real data. "Memory" is (13) solved self-consistently; w_0, w_a are CPL-equivalent values at $a = 1$. For the agegraphic model Ω_m is derived from n through $E(a = 1) = 1$, so n is its only cosmological parameter.

Conclusion. One QRC-motivated causal-memory dark-energy model is consistent with current supernova, BAO and chronometer data at the level of ΛCDM and has the DESI-preferred sign of drift. The data neither prefer nor exclude it, and its physical origin is open.

Limitations. No CMB likelihood was used, so the early-time behaviour of both dark-energy laws and the low $\Omega_m \approx 0.26$ of the agegraphic fit are not checked against Planck. Chronometer errors are diagonal and omit correlated stellar-population systematics. DESI's compressed BAO assumes a near-fiducial cosmology, which matters most for $R_h = ct$. BIC uses N equal to all supernovae, which penalises extra parameters heavily. Quasar and lensing claims were not testable for lack of data.

A Reproducibility

Package layout. The directory `paper` holds this document's source and PDF, and `build.sh` compiles it with `pdflatex`. The directory `checks` holds the mathematical checks, with their saved output in `checks/expected_output`. The directory `checks/cosmology` holds the real-data harness, a README with data sources and SHA-256 sums, and the result files of the run reported in Section 8.

Mathematical checks. Each needs Python 3 with numpy and scipy (tested with Python 3.13.9, numpy 2.4.2, scipy 1.16.3), reads no files and writes nothing:

```
cd checks
python3 -I -B check_gate1_selection.py   # 47 tests, about 1 s
python3 -I -B check_gate_results.py      # 26 tests, about 1 s
python3 -I -B check_gate5_family.py      # 18 tests, under 1 s
```

Each prints one line per test with the observed and expected outcome and exits with status 0 only if every outcome matches. Of the 91 tests, 35 run on deliberately wrong inputs and are expected to fail.

Test quality. Hand-made mutants of `check_gate1_selection.py` (sign of h flipped, transversality or span injectivity ignored, λ_1 and λ_2 swapped, groupoid weight dropped, tail bound weakened, matching algorithm or dynamic programme broken) were each detected. Line coverage (coverage.py 6.5) is 99 percent for the selection check and 100 percent for the other two. Three survived, each equivalent on the tested inputs: skipping the tail check, accepting any lossless rank-three configuration as the complement matching (it is the only transverse one), and ignoring the tail in the gap. No automated mutation tool was run.

Cosmology harness. Download the public files listed in `checks/cosmology/README.md` into a directory `DATA`, then

```
cd checks/cosmology
python3 -I -B validate.py   --data DATA   # about 12 s; exit 0 only if all pass
python3 -I -B run_models.py --data DATA   # about 200 s; refuses to run unless
                                          # validation passed
```

The data files are not redistributed in the package.

Repository documents. The theory statements are taken from the following files in `docs/` of the author's QRC repository: `QRC_CAUSAL_IMPEDANCE_GATE_FORMULATION.md` (foundational statement, gates); `QRC_PROJECT_STATUS.md` (current integrated position, 2026-11-19); `QRC_SPRE_AUTHORITATIVE_NESTED_RECIPROCITY_SOURCE_AND_COORDINATE_FREE_GATE1_THEOREM.md`; `QRC_GATE1_PARAMETER_FREE_NESTED_RECIPROCITY_AND_CUBIC_DIFFUSIVE_PHASE_THEOREM.md`; `QRC_GATE1_CANONICAL_NESTED_SOURCE_SOLE_Q_EXPORT_BINDER_THEOREM.md`; `QRC_GATE1_FINAL_COORDINATE_FREE_RELATIONAL_PHASE_THEOREM.md`; `QRC_GATE1_REFINEMENT_STATIONARY_HAAR_RESPONSE_AND_COORDINATE_FREE_DS3_CLOSURE_THEOREM.md`; `QRC_GATE5_INTERNAL_COMMUTANT_STANDARD_MODEL_AND_PHASE_THEOREM.md`; `QRC_GATE5_UNIFIED_GAUGE_PHASE_AND_PAIR_DUAL_ANOMALY_SELECTOR_THEOREM.md`; `QRC_GATE5_FINAL_INTERNAL_GROUP_ANOMALY_AND_JOINT_PHASE_FRONTIER.md`; `QRC_END_TO_END_GATE_TRAVERSAL_AND_PAIRWISE_REPAIR_LEDGER.md` (2026-11-18 audit). The audits of Section 7 are in `audits/2026-11-22_noncosmo`, `audits/2026-11-22_code` and `audits/2026-11-22_real_data` of the same repository.

Not verified for this preprint. The Gibbs concentration constant (Theorem 4.7) and the joint source and heat limit were checked by reading the argument, not by computation. The spin-bordism global-anomaly statement is cited. The horizon shear-response coefficient quoted in correction 1 (used only to show that the withdrawn step was inconsistent with its own starting action) is a standard result cited from the literature and was not re-derived by the audit. The refinement-stationary v2 contract was not reviewed.

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